

LECH KRUS¹, IRENA WORONIECKA-LECIEJEWICZ²MONETARY-FISCAL GAME ANALYZED USING
A MACROECONOMIC MODEL FOR POLAND

1. INTRODUCTION

The paper deals with the choice of policy mix in the context of mutual decision conditioning between the fiscal authority (the government) and the monetary authority (the central bank). Mathematical modeling, game theory and multicriteria optimization methods are applied. The policy mix means a combination of a monetary and a fiscal policy with a given restrictiveness/expansiveness level of each of them.

There exists a relatively rich bibliography dealing with interactions of fiscal and monetary policies. Blinder (1983) and after him Bennett, Loayza (2001) considered a simplified monetary-fiscal game with the fiscal and monetary authorities as players having respectively two fiscal and two monetary strategies: restrictive and expansive ones. The authors shown that an independent actions of the authorities may lead to the Nash equilibrium which is not Pareto optimal. They presented a similar interpretation relating to the prisoner's-dilemma problem and similar arguments for coordination of the policies. Nordhaus (1994) analyzed the problem of independence versus coordination of fiscal and monetary policies using a monetary-fiscal game. The game is based on a simple hypothetical macroeconomic model with utility functions of the government and of the central bank, dependent on their policy instruments. He presented an extended discussion relating to the Nash equilibria, Pareto optimality of payoffs, possible conflicts of interests of the authorities and suggestions for policy coordination. The Nordhaus game model is a starting point for further research. In the monograph (Marszałek, 2009, p. 131–132) a list of selected game models describing relations between the government and the central bank is presented and the models are characterized. Dixit, Lambertini (2001) considering a monetary-fiscal game underlined the importance of players credibility and fiscal discipline for results of the game. Lambertini, Rovelli (2003) continued the above research comparing the Nash and Stackelberg games. Many authors discuss and explain the facts that solutions in the above models of noncooperative monetary-fiscal games are not optimal and lead to a suboptimal policy mix, see for example the papers (Darnault, Kutos, 2005;

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Wojtyna, 1996; Marszałek, 2009). There are also some papers discussing policy mix problems, formulating arguments for policy coordination and presenting interesting results with the use of statistical data for Poland (Darnault, Kutos, 2005; Stawska, 2014). Libich et al. (2015) presented analysis and comparison of selected countries in so called *monetary vs fiscal leadership space*. Poland is located in the central part of the space. It means that in the case of Poland the fiscal authority does not dominate the monetary authority and nor vice versa. Cevik et al. (2015) examined the interactions between fiscal and monetary policy for some former transition, emerging European economies, also for Poland, over the 1995–2010 period by using a Markov regime-switching model. Empirical results suggest that monetary and fiscal policy rules exhibit switching properties between active and passive regimes. Libich, Nguyen (2015) analyzed strategic interaction between the central bank and government in the post global financial crisis period of 2010–2014. They concentrate on inflation targeting and its possible effect on both monetary and fiscal outcomes. Analysis of monetary and fiscal policies in Poland was also presented in OECD Economic Surveys (Monetary and fiscal policies to head off overheating, Poland 2008).

There are no publications dealing with interactions of the fiscal and monetary policies analyzed with the use of computational game models for Poland. The research presented in this paper tries to cover this gap. It is the first presentation of results within the monetary-fiscal games based on the macroeconomic model for Poland.

2. SUBJECT OF THE PAPER

This paper presents current results of the research carried on within the game theory, macroeconomic modeling and optimization methods applied for analysis of the policy mix problem. We try to analyze an efficiency of decisions made by the authorities, considering the Nash equilibria and Pareto optimality of their decisions. We try also answer the questions: how priorities of the monetary and fiscal authorities relate to the choice of the authorities' strategies; when and under what conditions the independent choice of strategies by the monetary and fiscal policies leads to the decisions which are economically effective and when a coordination of the decisions is required.

A noncooperative game called the monetary-fiscal game is formulated and analyzed in which the fiscal and monetary authorities play roles of players. Strategies of the monetary authority relate to the monetary policies with different restrictiveness/expansiveness level and are characterized by the real interest rate. Similarly, strategies of the fiscal authority mean the budget policies with different restrictiveness/expansiveness level. They are characterized by the budget deficit in relation to the GDP. The level of restrictiveness of each policy is defined by a value of the respective policy instrument. Each authority tries to obtain his respective economic target: a desired value of the GDP dynamics in the case of the fiscal authority, and a desired value of inflation in

the case of the monetary authority. It is assumed that the authorities make decisions independently.

A macroeconomic model for the Polish economy has been formulated on the basis of the New Neoclassical Synthesis concept. It includes four fundamental equations referring to the output gap, inflation, expected inflation and the Taylor rule of the interest rate. It allows analyzing of the economic situation in time. It takes into account the influence of interest rate on economy. The classical form of the model has been extended to include the influence of the fiscal policy. The model has been estimated using quarterly time series of data for Poland from the period 2000–2014.

A computer-based system calculating results of the game has been constructed using the above model. A sequence of simulations have been made in which payoffs of the game were derived for alternative monetary and fiscal policies. This paper presents continuation of the research described in the previous papers of the authors (Kruś, Woroniecka-Leciejewicz, 2015; Woroniecka-Leciejewicz, 2015a, b, 2010, 2008, 2007).

This paper is organized as follows. The next section 3 presents mathematical formulation of the game. The proposed macroeconomic NNS-MFG model is described in section 4. Section 5 presents results of the model estimation and examples of simulation runs. Analysis of the proposed monetary-fiscal game are shown and discussed in section 6. Conclusions are in section 7.

3. MATHEMATICAL FORMULATION OF THE GAME

Relations between the fiscal authority and the monetary authority can be described by a noncooperative, static, deterministic game. It is a single stage, deterministic, non-zero sum, perfect information game played by the central bank and the government. Each player takes decision independently taking into account possible reaction of the counter player. The game is defined in the strategic form as follows:

- (i) There are two players $i = 1, 2$: the fiscal authority (the government) and the monetary authority (the central bank).
- (ii) For each player a set Ω^i of pure strategies is defined. The strategies of the fiscal authority are those of the budgetary policy – from the extremely restrictive to the extremely expansive. The measure, denoted by b , of the degree of restrictiveness/expansiveness of the fiscal policy is constituted here by the level of budget deficit in relation to GDP. The strategies of the monetary authority range from the extremely restrictive one to the extremely expansive. The degree of restrictiveness/expansiveness is equivalent simply to the value of the real interest rate and denoted by r . Let Ω denote the Cartesian product of the sets of the strategies $\Omega = \Omega^1 \times \Omega^2$.
- (iii) For each player $i = 1, 2$, a function $h^i: \Omega \rightarrow \mathbf{R}$ is given defining outcome of the player i for given strategies undertaken by the both players. The outcome of the fiscal authority is measured by the GDP growth rate, denoted by y , where

$y = h^1(b, r)$. In the case of the monetary authority it is the inflation value, denoted by p , where $p = h^2(b, r)$. The functions $h^i, i = 1, 2$, are defined by the model relations.

- (iv) For each player $i = 1, 2$, a preference relation is given in the set of the attainable outcomes. It is assumed here that each authority tries to achieve a given goal: the fiscal authority – a desired value of GDP growth, the monetary authority – a desired value of inflation.

Outcomes of the game in the discrete form are presented in table 1. Payoffs in the table are denoted in the following manner: y_{ij} – payoff of the fiscal authorities (GDP growth rate) in the case where the government applies the fiscal strategy F_i and the central bank applies the monetary strategy M_j ; p_{ij} – cost to the monetary authorities (inflation) for the same pair of policies. The symbol b_i denotes the budgetary deficit in relation to GDP, corresponding to the i -th fiscal strategy, while r_j denotes the real interest rate, ascribed to the j -th monetary strategy.

It is assumed that the fiscal and monetary authorities take decisions independently, and the Nash equilibrium state in such a game is identified with the choice of a given combination of the budgetary and monetary policies.

Table 1.

The monetary-fiscal game – table of payoffs

Strategies		Central bank – the monetary policy			
		← restrictive		expansive →	
		Monetary strategy M_1 (interest rate r_1)	Monetary strategy M_2 (interest rate r_2)	...	Monetary strategy M_n (interest rate r_n)
Government – fiscal policy ↑ restrictive ↓ expansive	Fiscal strategy F_1 (budgetary deficit b_1)	y_{11} / p_{11}	y_{12} / p_{12}	...	y_{1n} / p_{1n}
	Fiscal strategy F_2 (budgetary deficit b_2)	y_{21} / p_{21}	y_{22} / p_{22}	...	y_{2n} / p_{2n}
	
	Fiscal strategy F_m (budgetary deficit b_m)	y_{m1} / p_{m1}	y_{m2} / p_{m2}	...	y_{mn} / p_{mn}

Source: Woroniecka-Leciejewicz (2010, p. 191).

4. MACROECONOMIC NNS-MFG MODEL

This section describes a recursive macroeconomic model based on the New Neoclassical Synthesis (NNS) concept. The model – called NNS-MFG has been constructed to analyze the discussed monetary-fiscal game (MFG).

Development of macroeconomic modeling based on dynamic stochastic general equilibrium concepts is observed in the last years, trying to find a consensus among alternative theoretical views on the key macroeconomic phenomena and modeling problems. The models proposed describe among others a temporary influence of the monetary policy on economic activity. The New Neoclassical Synthesis theory tries to combine positives of concurrent modern theories. It adopts the concepts of inter-temporal optimal behavior of households and firms, rational expectations and permanently balanced markets from the New Classical Economics and Real Business Cycle schools. On the other hand it accepts the assumption of monopolistic competition taken from the New Keynesian Economics school. This theoretical concept is called in literature as the New Neoclassical Synthesis (Goodfriend, King, 1997), the Neo-Wicksellian Model (Woodford, 2003), the New Keynesian Model (Blanchard, 2009), the new consensus in macroeconomics (Arestis, 2009), the New Keynesian macroeconomics (Spahn, 2009). Theoretical backgrounds of the NNS concept, discussion of doubts and proposals of possible extensions are presented by Bludnik (2010). The NNS models are constructed around three relations having deep roots in the economic theory and treated as essential in description of transmission of the monetary policy impulses. The three relations refer to the IS curve, the New Keynesian Philips curve and the Taylor rule. The basic NNS model is formally presented in the papers (Goodhart, 2007, p. 4; Galí, 2009, p. 2–3).

The proposed macroeconomic model NNS-GMF has been constructed to derive payoffs of the monetary-fiscal game in simulation experiments. The model has to fulfill the following prerequisites – it should enable analysis of the impact of the monetary and fiscal policies and their instruments: the real interest rate and the budget deficit in relation to GDP on the state of economy, i.e. on the GDP growth and inflation. It should be a dynamic model, enabling observation of the economic activity in time.

According to the ideas of the control theory the players strategies are input variables for which the state of economy and payoffs treated as outputs of the model are derived in recursive calculations. Therefore the real interest rate and the budget deficit in relation to GDP are exogenous variables. On the other hand the model is constructed on the basis of the NNS model (New Neoclassical Synthesis, Goodhart, 2007; Galí, 2009), which includes three key relations describing mechanisms of transmission of impulses of the monetary policy, i.e. the IS curve (equation of the demand gap), the Philips curve (the inflation equation) and the Taylor rule. The NNS model enables observation of the economic activity in time and takes into account the influence of the interest rate on the economy. The proposed NNS-MFG differs from the classic NNS model. Using the model one can observe not only effects of the monetary policy instruments but also instruments of the fiscal policy. For this reason it takes into account the budget expenditures also. The production gap equation has an analogic form as the equation in the basic NNS model, however the explanatory variables have no anticipative character. This simplification is typically applied in empirical studies, see for example Batini, Haldane (1999), Muinhos (2001), Freitas, Muinhos (2001), Kokoszcyński

et al. (2002). Because of the recursive calculation requirement a delayed variable is used in the place of the variable expressing the expected output gap.

The papers Budnik et al. (2009), Greszta et al. (2012) refer to the model NECMOD applied in NBP. It is relatively large and advance model constructed to prepare forecasts of main macroeconomic categories – first of all inflation, but also GDP and its components as well as other quantities important to pursue an effective and responsible monetary policy. The NNS-MFG is relatively simple and has been constructed not to prepare so accurate macroeconomic forecasts but to analyze the discussed monetary-fiscal game, especially interactions of monetary and fiscal strategies in the game as well as impact of targets assumed by the authorities on the game solution concepts.

Equations of the recursive model are presented below. Notation in the equations is assumed according to the basic NNS models.

Equation of the output gap

The equation, referring to the dynamic, inter-period version of the IS curve, describes an aggregated demand as the result of the optimal decisions made by a representative consumer. It has the following form:

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 (r_t - \pi_t^e - r_t^n) + \alpha_3 g_t, \quad (1)$$

where $x_t = y_t - y_t^n$, $g_t = G_t - G_t^n$.

The output gap x_t is defined as the difference of the current real production y_t and its natural level y_t^n in the equilibrium state with the perfectly elastic prices. The production is measured by the real Gross Domestic Product (GDP). A current value of the production gap depends on its delayed value and on the interest rate gap, where the interest rate gap is defined as the difference of the real interest rate and its natural level r_t^n . The real interest rate is calculated as the difference: the nominal interest rate r_t (WIBOR 1M) minus the expected inflation π_t^e .

The proposed model takes additionally into account in the first equation effects of the fiscal policy – an influence of the real budget expenditure G_t in the gap category, i.e. as the deviation from its natural value G_t^n . The natural levels of the product y_t^n , of the interest rate r_t^n and the budget expenditure G_t^n have been calculated using the Hodrick–Prescott filter. The production gap and the budget expenditure gap are defined in two versions as the absolute deviation from the natural value.

Inflation equation

The equation is known as the New Keynesian version of the Phillips curve. It presents a function of the aggregated supply based on price decisions of firms in the conditions of imperfect competition (Calvo, 1983). Inflation depends on the expected inflation π_t^e and on the output gap x_t . The equation has the form:

$$\pi_t = \beta_0 + \beta_1 \pi_{t-1}^e + \beta_2 x_t. \quad (2)$$

Equation of the expected inflation

The expected inflation is explained by its delayed value and by the current inflation. The equation has the form:

$$\pi_t^e = \delta_0 + \delta_1 \pi_{t-1}^e + \delta_2 \pi_t. \quad (3)$$

Equation of the interest rate (Taylor rule)

The equation describes a rule deriving the nominal interest rate by the central bank. The central bank derives the nominal interest rate in reaction on the deviation of inflation from the target π_t^* and on the current economic situation measured by the production gap. In this model it is the inflation target assumed by the National Bank of Poland in the Monetary Policy Guidelines. The equation describes reaction of the central bank according to the Taylor rule and has the form:

$$r_t = \varphi_0 + \varphi_1 r_{t-1} + \varphi_2 (\pi_{t-1} - \pi_{t-1}^*) + \varphi_3 x_{t-1}. \quad (4)$$

The delays are introduced in the equation similarly as in the previous equations. The delays relate to the difference of the current inflation and the target and to the output gap. They are introduced because of the recursive use of the model.

5. MODEL ESTIMATION

The above NNS-MFG model including the equations (1–4) has been estimated as a system of simultaneous equations using the Three-Stage Least Squares Method (3SLS) in the econometric GRETL package. Time series for the Polish economy from the period 2000–2014 (quarterly data) have been used in estimation. Time series used for estimation include relatively long time with the phases of strong and weak policy mix.

The statistical data have been collected from the following sources: the Central Statistical Office of Poland, the National Bank of Poland (NBP), the Ipsos group. Names and description of the exogenous and endogenous variables used for estimation are presented in table 2.

The interest rates WIBOR 1M and WIBOR 3M were considered in the model construction. The interest rate WIBOR 1M was finally assumed according to the research results indicating a stronger reaction of the WIBOR rates on the basic NBP interest rate for shorter maturity times and a lower reaction for the longer times (Janecki, 2012). On the other hand we obtained better estimation results for WIBOR 1M than for WIBOR 3M.

Results of estimation (model variant 1) are as follows (GRETL outputs; standard deviations for the estimated coefficients are given in brackets):

Table 2.

The variables used in the model estimation

Variable	Description
output_gap (endogenous)	The output gap (denoted by x in equations (1–4)) is defined as the difference between the real GDP and the natural level of output presented by the Central Statistical Office (GUS) in time series according to the principles of the “European System of National and Regional Accounts” (ESA); GDP in constant prices. The natural level is calculated as a long term trend of the of GDP using the Hodrick–Prescott filter.
output_gap_1	The output gap, one period delayed
Inflation (endogenous)	Inflation (π) is calculated on the basis of the consumer price index, analogic period of the previous year = 100 (GUS data).
expected_infl (endogenous)	Expected inflation is measured as the average inflation level expected in the next year (NBP, Ipsos data)
expected_infl_1	Expected inflation, one period delayed
WIBOR (exogenous)	The interest rate WIBOR 1M, nominal, at the beginning of each period (data from Money.pl (http://www.money.pl/))
WIBOR_1	The interest rate WIBOR 1M (nominal), one period delayed
WIBOR_gap (exogenous)	The interest rate gap (r^n) is the difference of the real interest rate (WIBOR 1M) and the natural rate, while the real interest WIBOR 1M is derived as the difference of the nominal rate WIBOR (r) and expected inflation (π^e). The natural (real) interest rate is calculated as a long term trend of the real interest rate using the Hodrick–Prescott filter.
expend_gap (exogenous)	The gap of the expenditure (g) of the public sector means deviation of the real public expenditure (G) from its natural level (G^n). The natural expenditure is calculated as a long term trend of the real public expenditure using the Hodrick–Prescott filter.
infl_target_dif (exogenous)	The difference between inflation (π) and the inflation target (π^*) indicated by NBP (Monetary Policy Guidelines data)
infl_target_dif_1	The difference between inflation and the inflation target, one period delayed

Source: own elaboration.

Equation 1

$$\text{output_gap} = 0.0273 + 0.6938 \text{ output_gap_1} - 0.4243 \text{ WIBOR_gap} + 0.1376 \text{ expend_gap}$$

(0.1195) (0.0840) (0.1247) (0.0621)

Equation 2

$$\text{inflation} = 0.5550 + 0.7532 \text{ expected_infl_1} + 0.3384 \text{ output_gap}$$

(0.1893) (0.0578) (0.0911)

Equation 3

$$\text{expected_infl} = -0.1963 + 0.1431 \text{ expected_infl_1} + 0.9152 \text{ inflation}$$

(0.1028) (0.0587) (0.0742)

Equation 4

$$\text{WIBOR} = 0.2653 + 0.9227 \text{ WIBOR_1} + 0.1967 \text{ infl_target_dif_1} + 0.2297 \text{ output_gap_1}$$

(0.0946) (0.0132) (0.0290) (0.0346)

Table 3.

Model, equation system, GRETl outputs

Equation 1: Estimation 3SLS, observations 2001:1–2014:4 (N = 56)

Dependent variable (Y): output_gap

Instruments: const output_gap_1 WIBOR_gap expend_gap expected_infl_1 WIBOR_1 infl_target_dif_1

Variable	Coefficient	Std. error	z	p-value
const	0.0272869	0.119475	0.2284	0.8193
output_gap_1	0.693819	0.0840444	8.255	1.51e-016
WIBOR_gap	-0.424272	0.124742	-3.401	0.0007
expend_gap	0.137646	0.0620940	2.217	0.0266

Mean dependent var -0.113802 S.D. dependent var 1.541290

Sum squared resid 46.33381 S.E. of regression 0.909610

R-squared 0.647118 Adjusted R-squared 0.626760

Equation 2: Estimation 3SLS, observations 2001:1–2014:4 (N = 56)

Dependent variable (Y): inflation

Instruments: const output_gap_1 WIBOR_gap expend_gap expected_infl_1 WIBOR_1 infl_target_dif_1

Variable	Coefficient	Std. error	z	p-value
const	0.554993	0.189279	2.932	0.0034
expected_infl_1	0.753164	0.0577744	13.04	7.61e-039
output_gap	0.338427	0.0911186	3.714	0.0002

Mean dependent var 2.606757 S.D. dependent var 1.678949

Sum squared resid 36.97738 S.E. of regression 0.812595

R-squared 0.765484 Adjusted R-squared 0.756635

Equation 3: Estimation 3SLS, observations 2001:1–2014:4 (N = 56)

Dependent variable (Y): expected_infl

Instruments: const output_gap_1 WIBOR_gap expend_gap expected_infl_1 WIBOR_1 infl_target_dif_1

Variable	Coefficient	Std. error	z	p-value
const	-0.196344	0.102801	-1.910	0.0561
expected_infl_1	0.143050	0.0587171	2.436	0.0148
inflation	0.915195	0.0741551	12.34	5.41e-035

Mean dependent var 2.586357 S.D. dependent var 1.736927

Sum squared resid 11.16849 S.E. of regression 0.446584

R-squared 0.936581 Adjusted R-squared 0.934188

Equation 4: Estimation 3SLS, observations 2001:1–2014:4 (N = 56)

Dependent variable (Y): WIBOR

Instruments: const output_gap_1 WIBOR_gap expend_gap expected_infl_1 WIBOR_1 infl_target_dif

Variable	Coefficient	Std. error	z	p-value
const	0.265344	0.0946242	2.804	0.0050
WIBOR_1	0.922672	0.0131957	69.92	0.0000
infl_target_dif	0.196669	0.0290075	6.780	1.20e-011
output_gap_1	0.229699	0.0345724	6.644	3.05e-011

Mean dependent var	5.858571	S.D. dependent var	3.643124
Sum squared resid	8.191705	S.E. of regression	0.382466
R-squared	0.988783	Adjusted R-squared	0.988136

Source: own elaboration.

Detailed estimation results for the NNS-MFG model are presented in the table 3 (GRETl package, system of equations, 3SLS). The estimation results show an acceptable goodness of fit. All the variables are statistically significant. The R-squared values are greater than 90% in the case of equations 3 and 4. The worse estimation has been obtained in the case of equation 1 and 2 with R-squared values: 63% and 76% respectively. However also in this case all the variables are statistically significant. The estimation inaccuracy is caused by model simplifications. The model describes influences of the economic policies only, it does not describe any influence of exogenous factors in the explicit form. Figure 1 presents matching of the endogenous variables: the theoretical values calculated by the estimated model compared to the empirical values.

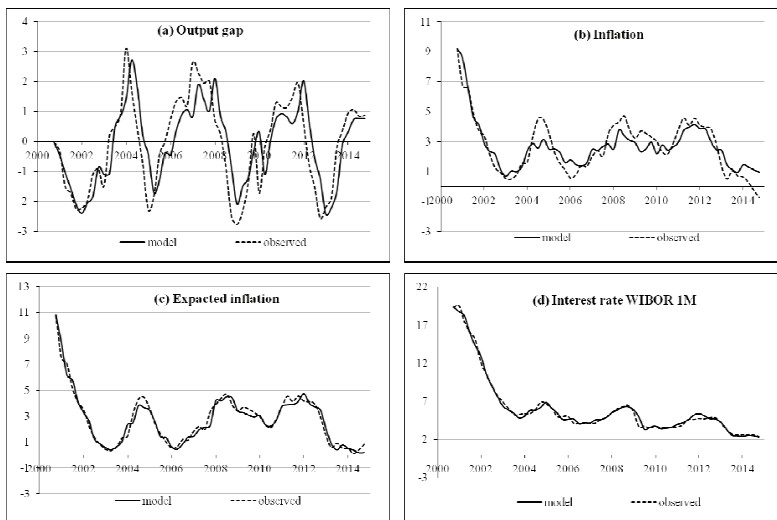


Figure 1. Estimated (3SLS) and observed values of the variables of the model: (a) output gap, (b) inflation, (c) expected inflation, (d) interest rate WIBOR 1M

Source: own elaboration.

6. ANALYSIS OF THE MONETARY-FISCAL GAME

Strategies, outcomes and payoffs of the game were analyzed using the NNS-MFG model presented in section 4 for the estimation results shown in section 5. Computer simulations have been made for different variants of the model parameters and different initial values of the model variables. Selected simulation results are presented.

Simulation assumptions

The initial state of the economy is represented by the model variables on the basis of the empirical data in the last quarter of 2000. The model variables are calculated using the NNS-MSG model since the first quarter of 2001, while the nominal interest rate is calculated by the Taylor rule and the public expenditure gap according to the statistical data on the real public expenditure. In a selected period (in the presented results: 8 quarters since 1-st quarter of 2008) an impulse changing the policy mix is introduced. The instruments of the policy mix, i.e. real interest rate and the budget deficit in relation to GDP are assumed on a given constant level in this period of time. The nominal interest rate is calculated as the real rate plus expected inflation. The real public expenditure and the budget expenditure gap are calculated on the basis of the budget deficit and the tax rate. After the time the real interest rate is derived according to the Taylor rule and the budget deficit to GDP ratio – according to the ex post data.

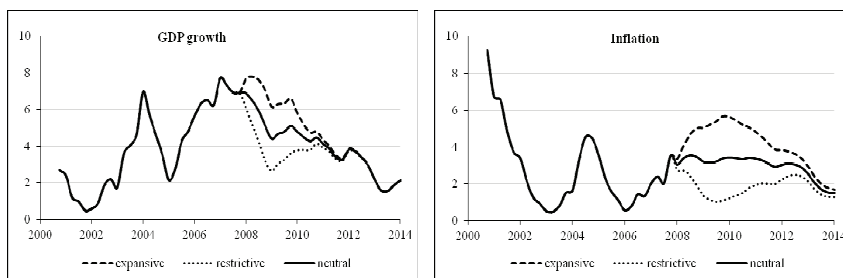


Figure 2. GDP growth rate and inflation in 2000–2014 simulated for the three variants of policy mix: expansive, restrictive and neutral

Source: own elaboration.

Figure 2 illustrates dynamic effects of the policy mix changed in the considered period of time. Three variants of the policy mix in this period are compared: a policy more expansive than the policy historically implemented (real interest rate was assumed 1 percent points lower and the budget deficit in relation to GDP – 1 percent point greater than the historical values), a policy more restrictive than the historical policy (real interest rate was assumed 1 percent points greater and the budget deficit in relation to GDP – 1 percent point lower than the historical values), and a neutral policy, when the instruments were assumed on the historical level. It can be observed that effects of the introduced changes of the policies are temporary, shorter in the case

of the GDP growth and longer in the case of inflation. The more expansive policy mix results in a greater GDP growth and in a greater inflation in comparison to the neutral path. The effects of the more restrictive policy are reverse.

Payoffs of the monetary-fiscal game being effects of the changed policy mix are measured by the average annual production growth (denoted as y in the game formulation in section 3) and by average annual inflation (denoted by p) in the period of 8 quarters since the changes of the policy mix have been introduced. The payoffs were calculated using the NNS-MFG model relations for the interest rate (r) and budget deficit in relation to GDP (b) treated as exogenous variables in the considered period of time. The functions $h^1(b, r)$ and $h^2(b, r)$ introduced in the game formulation denote the dependence of the payoffs on the policy instruments due to the model relations.

Admissible values of the policies' instruments have been assumed in a form of intervals. The interest rate has been changed in the interval [6%, -1%] and the budget deficit in relation to GDP – in the interval [-1, 6%]. The computer-based system simulating the game and all calculations were made in the MSeXcel environment using the VBScript language and the embedded optimization solver. Selected results are presented and discussed below.

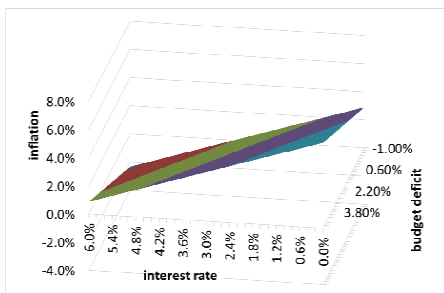


Figure 3. Outcomes of the monetary authority
Source: own elaboration.

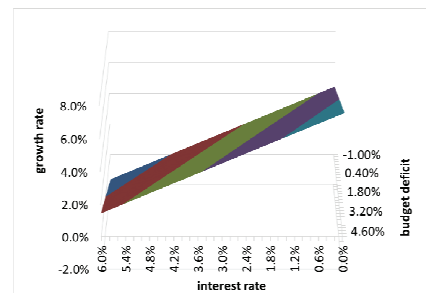


Figure 4. Outcomes of the fiscal authority
Source: own elaboration.

Figures 3 and 4 present the outcomes of the authorities, as dependent on assumed strategies. Inflation (figure 3) can be obtained on a low level when a restrictive monetary policy and a restrictive fiscal policy are applied. More expansive monetary and fiscal policies lead to an increase of inflation and of the economic growth. On the other hand more restrictive monetary and restrictive fiscal policies lead to a decrease of the economic growth (figure 4).

Let us assume that the monetary and fiscal authorities try to achieve given targets of their policies. Let the monetary authority assume the inflation goal on the level p^g , and let the fiscal authority try to achieve the GDP growth rate on the level y^g . Let Ω denotes the set of admissible pairs (b, r) of strategies. The best response strategies of the authorities can be obtained as solutions of the optimization problems:

Min $|h^1(b, r) - y^g|$ with respect to $b \in \Omega^1$ solved for all $r \in \Omega^2$, in the case of the fiscal authority and

Min $|h^2(b, r) - p^g|$ with respect to $r \in \Omega^2$, solved for all $b \in \Omega^1$, in the case of the monetary authority, respectively.

Examples of the best response strategies derived for different targets of the authorities are presented in figure 5. Figure 5, part (a) presents the best response strategies of the monetary authority for the three different targets: inflation = 2%, 2.5%, 3%, and the best response strategies of the fiscal authority for the target: GDP growth = 3.5%. Figure 5, part (b) presents the best response strategies of the fiscal authority for the three different targets: GDP growth = 3%, 3.5%, 4%. and the best response strategies of the monetary authority for the target: inflation = 2.5%. The Nash equilibria which are Pareto optimal in the assumed interval of the policies' instruments are shown.

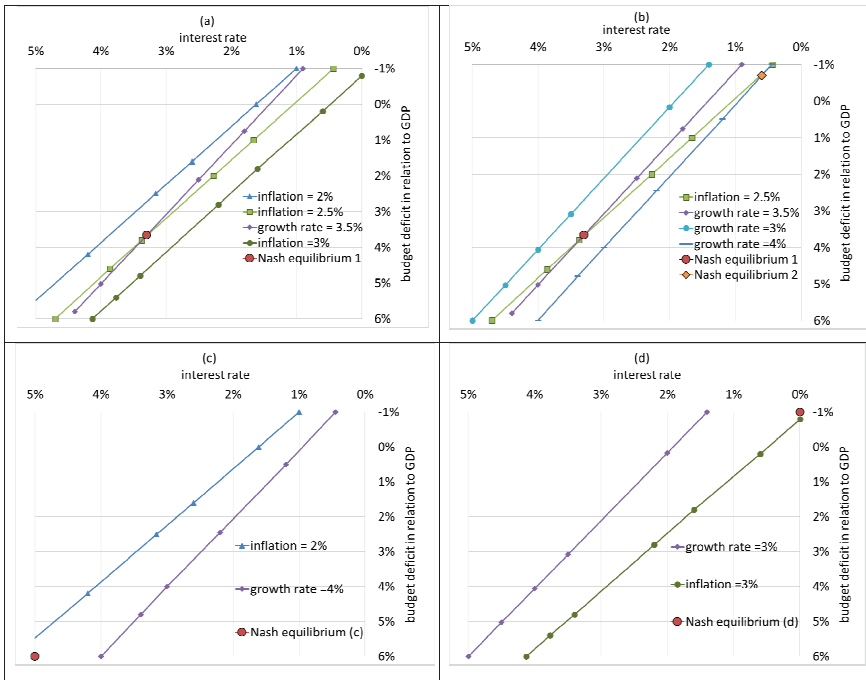


Figure 5. The best response strategies:
 (a) for different monetary targets and the fiscal target – growth rate = 3.5%;
 (b) for different fiscal targets and the monetary target – inflation = 2.5%;
 (c) for the targets: growth rate = 4% and inflation = 2%;
 (d) for the targets: growth rate = 3% and inflation = 3%

Source: own elaboration.

It can be observed how the level of restrictiveness/expansiveness of the monetary policy depends on the level of restrictiveness/expansiveness of the fiscal policy. A more expansive fiscal policy leads to a more restrictive monetary policy taken by the central

bank trying to limit inflation exceeding the inflation target. If the budget deficit is higher, then the required inflation is obtained for respectively higher interest rates. Analogously, if the government carries out a more restrictive budget policy, then the central bank will apply a less restrictive (more expansive) monetary policy with relatively lower interest rates.

On the other hand a more restrictive monetary policy causes in reaction a more expansive budget policy. If the interest rate is higher, then the required growth rate can be achieved by applying a more expansive fiscal policy supporting a higher growth rate. That means the government should assume a relatively greater budget deficit. Inversely the government can implement more restrictive fiscal policy limiting the budget deficit in reaction on a more expansive monetary policy.

The simulation results show how changes of the targets of fiscal and monetary policies influence on the best response strategies and on the Nash equilibrium state, i.e. on the choice of the respective policy mix. More ambitious target of fiscal policy with a high required economic growth causes that the best response budget strategy moves into more expansive one and vice versa in the opposite case. The higher inflation targets assumed by the monetary authority cause that the best response strategies of the central bank move into more expansive monetary policies. In the opposite case, of the lower inflation target, the best response monetary policy moves into a more restrictive one. Changes of the targets assumed by the fiscal and monetary authorities result in respective positioning of the Nash equilibrium.

There are two cases of the best response strategies in the considered game. The first when the best response strategies cross in the set of admissible strategies, and the second when they do not cross in this set. The cross point in the first case defines the Nash equilibrium. One can easily see that a deviation of any strategy from the point leads to a worse payoff of the respected player (Nash, 1951). In the second case the real Nash equilibrium is out of the set of admissible strategies. The theoretical Nash equilibrium in the set also exists but on the boundary of the set of admissible strategies. One can find also that a deviation from the point leads to worse payoffs of at least one of the players.

Figure 5 (c) illustrates the case when the fiscal and monetary authorities assume too ambitious targets in the given economic state. The monetary authority assumes the restrictive inflation target (2%) and the fiscal authority would like to achieve the high growth (4%). The best response strategies do not cross in the assumed intervals of the policies' instruments. The theoretical Nash equilibrium is at the most restrictive monetary and at the most expansive fiscal policy (the point: $r = 5\%$, $b = 6\%$ in the figure). It is not Pareto optimal. Another example of not ambitious targets is presented in figure 5 (d) for the soft inflation target = 3% and the not demanding fiscal target: GDP growth = 3%. The theoretical Nash equilibrium is in this case at the most expansive monetary and the restrictive fiscal policy (the point: $r = 0\%$ and $b = -0.9\%$ in the figure).

The results show that efficiency of the policy mix depends on the targets assumed by the monetary and fiscal authorities, therefore the respective coordination of the targets will be beneficial for both authorities.

The Nash equilibrium (Nash 1) in figure 5 describes the state which can be compared to the real state of the economy in Poland in the analyzed period of 8 quarters 2008:1–2009:4. Comparing the obtained results to the historical data (average annual growth rate = 3.3% and average annual inflation = 3.8%) in this period one can state that the calculated equilibrium state indicates possibility of policies giving better economic effects: a greater growth rate = 3.5% and lower annual inflation = 2.5%. More expansive policy mix, especially the fiscal policies could be applied with the real interest rate = 3.3%, close to the historical one equal to 3.4% on the average in the period 2008–2009, and with the budget deficit to GDP ratio = 3.8% greater than the historical one equal to 2.7% on the average in this period. It is an open question why the authorities did not carry out these possible better policies. The decision making process was in this time rather difficult, when negative effects of economic recession at the end of 2008 and in 2009 were observed. More general analysis of alternative policies mix in comparison to the historical ones carried out in Poland is included in a separate paper Kruś, Woroniecka-Leciejewicz (2016).

The obtained simulation results indicate a possibility of a case when the Nash equilibrium is Pareto optimal, but also a case when the Nash equilibrium leads to the solution not beneficial for one or both players. The last case is known in the literature as the prisoners' dilemma, when the policy coordination is desired. It is shown in the paper that the respective coordination of monetary and fiscal targets may lead to Pareto optimal Nash equilibria and effective policy mix.

7. CONCLUSIONS

This paper presents selected results of the research dealing with mutual interactions of the monetary and fiscal policies. The results have been obtained using the game theory and optimization methods. A dynamic macroeconomic model, called NNS-MFG model, has been formulated and estimated using the statistical data for Poland. A noncooperative monetary-fiscal game has been formulated in which payoffs of players – namely monetary and fiscal authorities are calculated using the model equations. The results for the monetary-fiscal game presented in this paper are the first obtained for Poland with the use of a macroeconomic model.

The macroeconomic NNS-MFG model describes influences of the instruments of the monetary and fiscal policies on the state of the economy, i.e. influences of the real interest rate and of the budget deficit in relation to GDP on the growth rate and inflation. It is based on concept of the New Neoclassical Synthesis model. It includes four equations describing the production gap, inflation, expected inflation and the Taylor rule.

The basic NNS model describes a transmission of the monetary policy impulses. In comparison to the basic NNS model, this model has been extended to describe influences of the fiscal policy. It takes into account the budget expenditure gap. The model parameters have been estimated using time series 2000–2014. The Three-Stage Least Squares method in the GRETl package has been used for the model treated as a system of simultaneous linear equations. The estimated model has been implemented in the form of a recursive algorithm in a computer-based system. The system calculates the payoffs of players and other variables of the model dependently on strategies implemented by the players. The system derives also the best response strategies dependently on the targets assumed by the monetary and fiscal authorities, as well as the Nash equilibria and the Pareto optimal outcomes.

A number of simulations has been made and the obtained results have been analyzed. The system derives detailed quantitative results, but also some qualitative conclusions can be formulated. There are some values of the targets assumed by the monetary and fiscal authority, for which the best response strategies cross in the interval of assumed admissible values of the instruments. The cross point relates to the Nash equilibrium and the equilibrium is Pareto optimal. However for some values of the targets the Nash equilibrium can be non Pareto optimal. For example in the case of too ambitious targets of the authorities the Nash equilibrium shifts into the most restrictive monetary policy and the most expansive fiscal policy what leads to non-effective non Pareto optimal solutions. The results show that in such cases a coordination of the monetary and fiscal policies is required.

Summarizing, the main result of the presented research consists in construction of a computer-based tool supporting analysis of the monetary fiscal game with the use of a respective macromodel estimated for Poland. More detailed results include the proposed and constructed NNS-MFG model, estimation of the model parameters using the statistical data for Poland, procedures calculating payoffs of the proposed monetary-fiscal game, formulation of respective optimization problems and procedures deriving the best response strategies of players, numerical results discussed above. The presented numerical results illustrate only selected features of the system.

The system may support looking for the Pareto-optimal consensus of the authorities in the policy mix problem. It can be checked when the targets assumed by the fiscal and monetary policies lead to the Pareto-optimal Nash equilibrium and the equilibrium can be derived. On the other hand one can check when the priorities of the monetary and fiscal authorities lead to non-effective Nash equilibria and when coordination of monetary and fiscal policies is required.

The constructed macroeconomic model is relatively simple, but the proposed approach can be applied also for more extended versions of the model. Such an extended nonlinear model describing the influence of the policy mix instruments on the economy in a more adequate way is planned. Further research include also analysis of the problem using dynamic game concepts when a sequence of decisions made by the authorities is considered. Another direction deals with development

of multicriteria optimization tools supporting analysis and consensus seeking. The methods of multicriteria bargaining support proposed in Kruś (2011, 2014) can be applied to construct such optimization tools.

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ANALIZA GRY MONETARNO-FISKALNEJ Z WYKORZYSTANIEM MAKROEKONOMICZNEGO MODELU DLA POLSKI

Streszczenie

W pracy sformułowano i przeanalizowano grę monetarno-fiskalną opisującą interakcje między władzami monetarnymi i fiskalnymi. Każdy z graczy stara się osiągnąć założony cel: władze monetarne – pożądaną poziom inflacji, władze fiskalne – pożądaną wzrost gospodarczy. Zbudowano model makroekonomiczny oparty na koncepcji nowej syntezy neoklasycznej. Model ten, zawierający cztery podstawowe równania: luki popytowej, inflacji, inflacji oczekiwanej i regułę Taylora, został rozbudowany z uwzględnieniem wydatków budżetowych. Dokonano estymacji modelu na dla gospodarki polskiej na podstawie szeregów czasowych 2000–2014.

Zbudowano system komputerowy wyznaczający wypłaty gry. Przeprowadzono szereg symulacji z wykorzystaniem zbudowanego modelu. Przedstawiono skutki polityk monetarnych i fiskalnych alternatywnych do realizowanych w przeszłości. Wyznaczono i przeanalizowano optymalne strategie odpowiedzi oraz strategie równowagi Nasha. Wskazano na przypadki, gdy równowagi Nasha są Pareto optymalne oraz takie, które prowadzą do wyboru skrajnie restrykcyjnych lub skrajnie ekspansywnych polityk, a uzyskiwana równowaga nie jest Pareto optymalna. Wskazana jest wtedy koordynacja polityk.

Słowa kluczowe: gra monetarno-fiskalna, model makroekonomiczny, policy mix, równowaga Nasha, Pareto optymalność

MONETARY-FISCAL GAME ANALYZED USING A MACROECONOMIC MODEL
FOR POLAND

A b s t r a c t

In the paper a monetary-fiscal game is formulated and analyzed. It describes interactions of the monetary and fiscal authorities. Each authority tries to achieve its own goal: the fiscal authority – assumed GDP growth, and the monetary authority – an inflation level. A macroeconomic model for the Polish economy has been formulated on the basis of the New Neoclassical Synthesis concept and respectively extended to describe effects of the fiscal instruments. The model parameters have been estimated using statistical data of the Polish economy from the period 2000–2014.

A computer-based system calculating results of the game has been constructed using the above model. A sequence of simulations have been made in which payoffs of the game were derived for alternative monetary and fiscal policies. Results of the policy mix strategies alternative to the historical policies in Poland were also considered.

In the paper the best response strategies of the authorities and the Nash equilibria are analyzed when the authorities assume independently their goals. The simulation results are presented and discussed for the cases when the Nash equilibria is Pareto optimal but also when it is not Pareto optimal. It is shown also that the best response strategies may lead to the extremely restrictive or expansive policies.

Keywords: monetary-fiscal game, macroeconomic model, policy mix, Nash equilibrium, Pareto optimality